Freshman Meet 1 – November 7, 2012
Round 1: Evaluation of Algebraic Expressions and Order of Operations

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Simplify and express as a fraction reduced to lowest terms:

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{6}\right)$$

2. Given the equation

$$-x^4+2x^3-3x^2+4x-k=0,$$

find k if $x=-3$.

3. If $x \star y = 5x+y$ and $x \circ y = 2xy + (x \star y)$, evaluate $(2 \circ 5) \star (3 \star 4)$.

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________

(3 pts.) 3. ____________________

Freshman Meet 1 – November 7, 2012
Round 2: Solving Linear Equations

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Solve for x: $\dfrac{x+5}{7} = 11 - x$.

2. Solve for y: $\dfrac{y}{5} - \dfrac{y-4}{2} = -3$.

3. Solve for z:

$$\frac{1}{2}z - \frac{1}{3}z + \frac{1}{4}z - \frac{1}{5}z + \frac{1}{6}z = z + 3.$$

ANSWERS

(1 pt.) 1. $x =$ ____________________

(2 pts.) 2. $y =$ ____________________

(3 pts.) 3. $z =$ ____________________

Freshman Meet 1 – November 7, 2012
Round 3: Logic Problems

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Three teachers teach one of three subjects: math, science, or history. Teacher A does not teach science, Teacher B does not teach history, and Teacher C teaches math. What does Teacher A teach?

2. From 11:59 AM to 11:59 PM of the same day, how many times do the hour hand and the minute hand of a clock meet?

3. I drive from Worcester to Boston at 60 mph and return at 40 mph. What is my average speed for the round trip?

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________

(3 pts.) 3. ____________________ mph

Freshman Meet 1 – November 7, 2012
Round 4: Ratio, Proportion, and Variation

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Some areas of California depend on the snow pack of the Sierra Nevada Mountains for their water supply. If 250 cm^3 of snow will melt to 30 cm^3 of water, how much water (in cm^3) does 900 cm^3 of snow produce?

2. A dear old Grandpa named Lunn
Is twice as old as his son
Twenty-five years ago
Their age ratio
Strange enough was three to one.

How old (in years) is Grandpa Lunn now?

3. The variable x varies directly as the square of y and inversely as z. If $x = 400$ when $y = 10$ and $z = 2$, find y when $x = 2$ and $z = 8$.

ANSWERS

(1 pt.) 1. ____________________ cm^3

(2 pts.) 2. ____________________ years

(3 pts.) 3. ____________________

Freshman Meet 1 – November 7, 2012
TEAM ROUND

All answers must either be in simplest exact form or rounded to EXACTLY three decimal places, unless stated otherwise. (3 POINTS EACH)

APPROVED CALCULATORS ALLOWED

1. In how many zeros does the base 10 representation of 2012! end?
2. If $a = 0.1$ and $b = -1/3$, evaluate
$$\frac{a-b}{a-b(b-a)}.$$
3. Angles K, D, and G are acute, right, and obtuse angles of different triangles, but not necessarily in that order. Angle D is not congruent to its supplement. Angle K is not the largest angle in its triangle. Classify the three angles.
4. If $f(x) = -x + 2$, $g(x) = 2x + 1$, and $h(x) = 3x^2 + 2$, find $g(f(h(2)))$.
5. If a hen and a half can lay an egg and a half in a day and a half, how many days does it take for three and a half hens to lay four and a half eggs?
6. One side of a square is increased by 12 cm and the other side is decreased by 8 cm. The area of the new rectangle is the same as the area of the original square. Find the length of the side of the square, in centimeters.
7. What integer can you multiply by 52,631,578,947,368,421 to give a product whose digits are all 9s?
8. Solve for x:
$$3\Big\{3 - 3\big[3 - (3 - 3x)\big]\Big\} = 0.$$

Freshman Meet 1 – November 7, 2012
TEAM ROUND ANSWER SHEET

1. ______________________________

2. ______________________________

3. Angle K: ______________________________

   Angle D: ______________________________

   Angle G: ______________________________

4. ______________________________

5. ______________________________ days

6. ______________________________ cm

7. ______________________________

8. ______________________________

Freshman Meet 1 – November 7, 2012
ANSWERS

ROUND 1
(Worcester Academy, Quaboag, Millbury)

1. 1/6 (only)
2. -174
3. 194

ROUND 2
(North, Auburn, Bartlett)

1. 9
2. $50/3 = 16\frac{2}{3}$
3. $-180/37 = -4\frac{32}{37}$

ROUND 3
(St. John's, Bancroft, St. John's)

1. history
2. 11
3. 48

ROUND 4
(Hopedale, Bromfield, Assabet Valley)

1. 108
2. 100
3. $\sqrt{2}$

TEAM ROUND
(QSC, Algonquin, Southbridge, Bromfield, Hudson, Shrewsbury, Leicester, Assabet Valley)

1. 501
2. $-39/4 = -9\frac{3}{4} = -9.75$
3. K: acute; D: obtuse; G: right
4. -23
5. $27/14 = 1\frac{13}{14} \approx 1.929$
6. 24
7. 19
8. $1/3 = 0.\overline{3} \approx 0.333$

Varsity Meet 1 – November 7, 2012
FULL SOLUTIONS

ROUND 1

1. We have

$$\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\frac{5}{6}=\boxed{\frac{1}{6}}.$$

2. Rearrange and then plug in the given value of x to find $k = -81-54-27-12 = \boxed{-174}$.

3. We have $(2\circ 5)\star(3\star 4) = (20+10+5)\star(3\star 4) = 35\star 19 = 175+19 = \boxed{194}$.

ROUND 2

1. $x = \boxed{9}$:

$$\begin{aligned}\frac{x+5}{7} &= 11-x\\ x+5 &= 77-7x\\ 8x &= 72\\ x &= 9\end{aligned}$$

2. $y = \boxed{50/3}$:

$$\begin{aligned}\frac{y}{5}-\frac{y-4}{2} &= -3\\ \frac{2y-5y+20}{10} &= -3\\ -3y+20 &= -30\\ 50 &= 3y\\ 50/3 &= y\end{aligned}$$

3. $z = \boxed{-180/37}$:

$$\begin{aligned}
\frac{1}{2}z - \frac{1}{3}z + \frac{1}{4}z - \frac{1}{5}z + \frac{1}{6}z &= z+3 \\
z\left(\frac{30-20+15-12+10}{60}\right) &= z+3 \\
\frac{23}{60}z &= z+3 \\
-3 &= \frac{37}{60}z \\
-\frac{180}{37} &= z
\end{aligned}$$

ROUND 3

1. Since Teacher C teaches math, then teachers A and B teach history and science. We are given that Teacher B does not teach history, so that means Teacher A does.

2. The minute hand makes 12 complete revolutions and the hour hand makes one. *From the point of view of the hour hand*, the minute hand makes 11 revolutions, so they meet $\boxed{11}$ times.

3. The distance does not matter; let's call it D. Let x be the average speed for the entire trip. Then, we can calculate the total time in two ways: the two separate directions and the round-trip:

$$\begin{aligned}
\frac{D}{60} + \frac{D}{40} &= \frac{2D}{x} \\
\frac{5D}{120} &= \frac{2D}{x} \\
5x &= 240 \\
x &= 48
\end{aligned}$$

The distance D drops out and we are left with the average speed $\boxed{48}$.

ROUND 4

1. Our proportion gives $\frac{900}{250} \cdot 30 = \frac{18}{5} \cdot 30 = \boxed{108}$.

2. Let Grandpa Lunn's age be x. Then, from the limerick, we have the ratio

$$\frac{x-25}{x/2-25}=\frac{3}{1}$$

Solving by cross-multiplication, we find $x=\boxed{100}$.

3. We are given that $x \propto \frac{y^2}{z}$, or that $\frac{y^2}{xz}=k$ for some constant k. Plugging in the given values, we find that $k=\frac{100}{800}=\frac{1}{8}$. Therefore, when $x=2$ and $z=8$, we have $\frac{1}{8}=\frac{y^2}{16}$ and $y=\boxed{\sqrt{2}}$.

TEAM ROUND

1. Trailing zeros, in base 10, represent factors of 10. Consider the prime factorization of 2012!. Clearly there are more factors of 2 than factors of 5, so we just need to count the number of factors of 5 (here we are using the prime factorization $10=2\cdot 5$).

   Every 5th integer contains a factor of 5, every 25th integer contains an *additional* factor of 5, every 125th integer contains a third factor of 5, etc.

   Therefore the number of factors of 5 in 2012! is $402+80+16+3=\boxed{501}$.

2. Plugging in, we have:

$$\begin{aligned}
\frac{a-b}{a-b(b-a)} &= \frac{\frac{1}{10}+\frac{1}{3}}{\frac{1}{10}+\frac{1}{3}(-\frac{1}{3}-\frac{1}{10})}\\
&= \frac{\frac{13}{30}}{\frac{1}{10}+\frac{1}{3}(-\frac{13}{30})}\\
&= \frac{\frac{13}{30}}{\frac{1}{10}-\frac{13}{90}}\\
&= \frac{\frac{13}{30}}{-\frac{4}{90}}\\
&= \boxed{-\frac{39}{4}}.
\end{aligned}$$

3. Angle D is not congruent to its supplement, so it is not a right angle. Angle K is not the largest angle in its triangle, so it must be acute. Therefore angle G must be right.

4. Evaluate from the inside out: $g(f(h(2)))=g(f(14))=g(-12)=\boxed{-23}$.

5. The number of hens h, the time t, and the number of eggs laid n are related by the following equation, where k is a constant to be determined:

$$kht = n$$

Plugging in the given values, $k \cdot \frac{3}{2} \cdot \frac{3}{2} = \frac{3}{2}$, so $k = \frac{2}{3}$. Therefore, for our problem, $\frac{2}{3} \cdot \frac{7}{2} t = \frac{9}{2}$, to $t = \boxed{\frac{27}{14}}$.

6. Let the side length of the square be x. Then, equating areas, we have $x^2 = (x+12)(x-8)$. Expanding the right side by FOIL, we have $x^2 = x^2 + 4x - 96$, or that $4x = 96$. Therefore $x = \boxed{24}$.

7. Call the integer x. The digits in 52,631,578,947,368,421 (possibly with leading zeros) must be part of a repeating decimal expansion for the fraction $1/x$ (think about the decimal expansions for fractions like $1/3$ or $1/7$). Therefore, enter as many digits of the number as possible into your calculator and take the reciprocal: $5263157895^{-1} = 1.9 \times 10^{-10}$, so the number we are looking for is $\boxed{19}$.

   [Interesting side-note: A result from number theory states that the period of a repeating decimal with denominator n is at most $n - 1$ (the numerator is irrelevant). For our case $n = 19$, the period of the repeating decimal is 18: 052631578947368421.]

8. $x = \boxed{1/3}$:

$$\begin{aligned} 3\Big\{3 - 3\big[3 - (3 - 3x)\big]\Big\} &= 0 \\ 3 - 3\big[3 - (3 - 3x)\big] &= 0 \\ 3 - 3(3x) &= 0 \\ 3 - 9x &= 0 \\ 1/3 &= x \end{aligned}$$